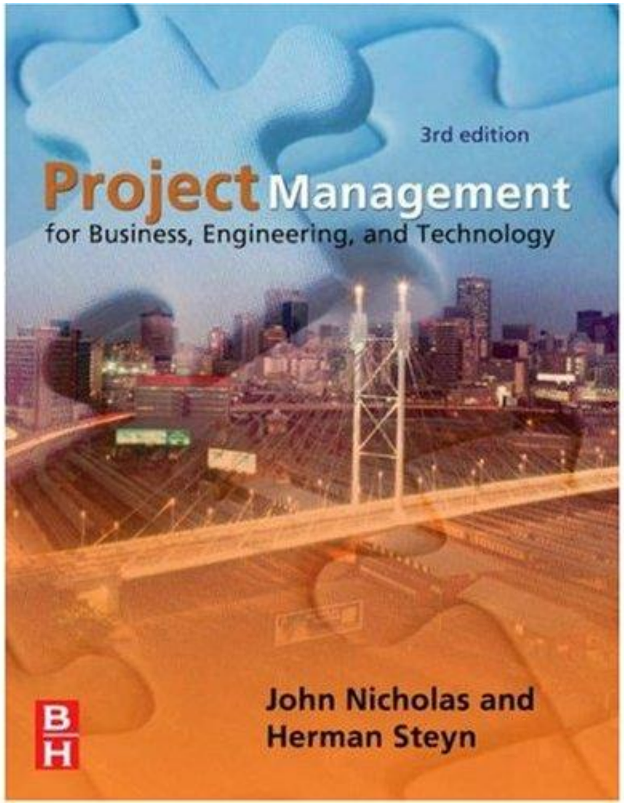
Chapter 7 (Cont'd)

PERT

Project Management for Business, Engineering, and Technology



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Variability of Activity Duration

Until now we considered estimates of activity duration to be “most likely” times: a single, deterministic value for each activity and, hence, for project completion time

Variability of Activity Duration

The duration of an activity is not fixed

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Consider the time it takes to drive to some destination

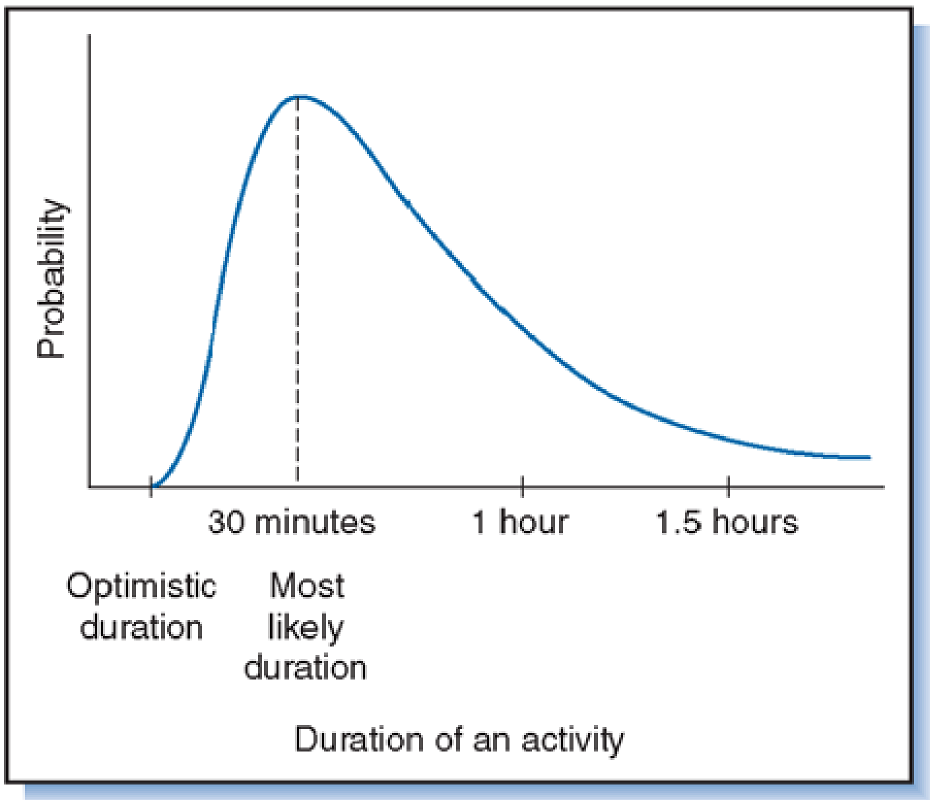


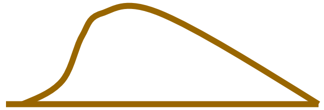
Figure 7-8
Variability of activity duration.

Project Duration –Variability of Activity Duration

Activity duration is not a single, deterministic value

There is a range of possible durations for most activities

The range of possible *activity* durations can be presented as a distribution curve:

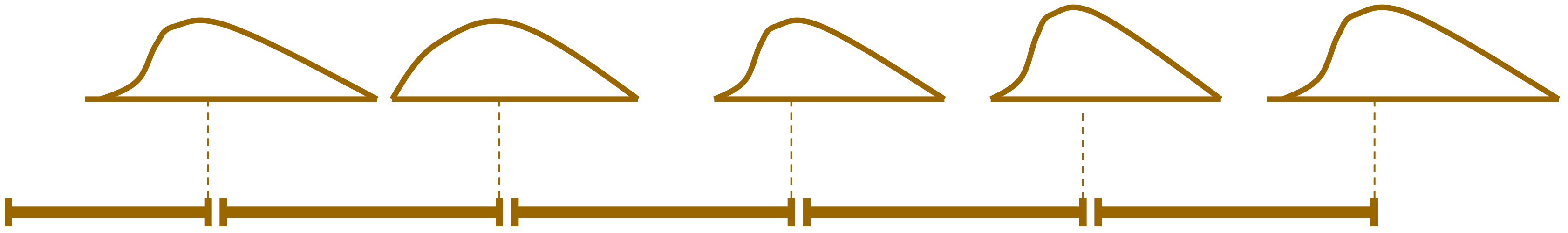


Now consider a *network* of activities …

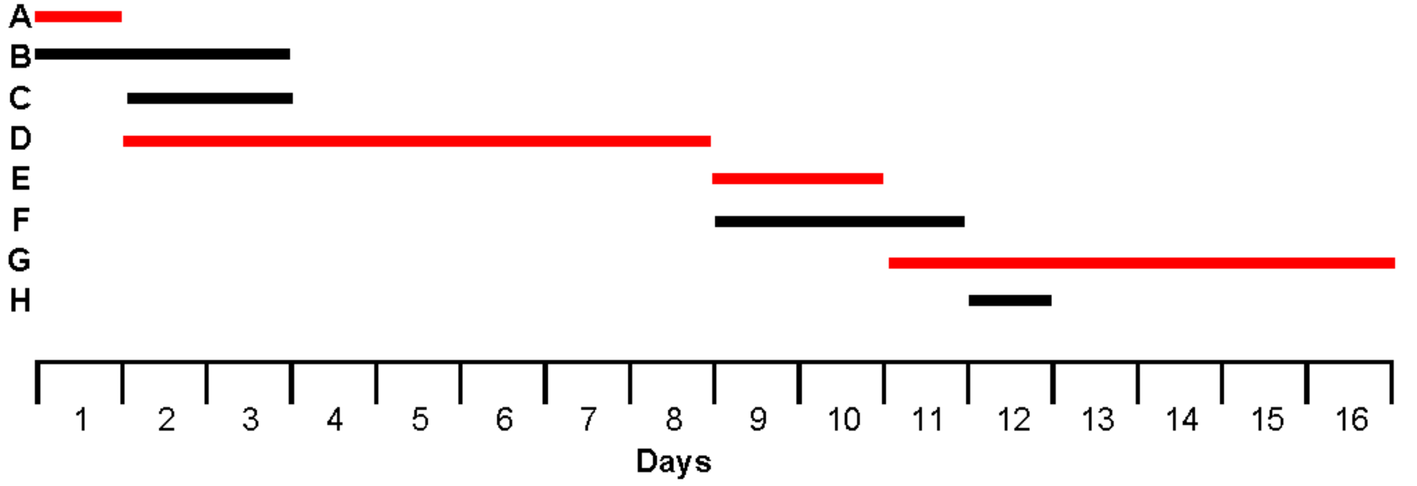
Project Duration –Variability of Activity Duration

Project duration is determined by the duration of activities on the critical path

But the duration of each activity is variable.
Each activity has a duration distribution:



Project Duration: Example



Longest path is A-D-E-G

So project duration is 16 days

But there is variability …

Project Duration

- In reality, actual activity times will vary, hence so will project completion time.
- Might say that, e.g., project will be completed in 16 days, but also acknowledge it will likely be completed earlier or later than that.

The PERT Technique
(Program Evaluation & Review Technique)

PERT:

Program

Evaluation

and Review

Technique

The PERT technique addresses variability of the duration of activities on the critical path

PERT (cont'd)

- PERT is a method that treats completion times as *probabilistic* (stochastic) events
- PERT was developed to deal with uncertainty in projects, and to estimate project duration when activity times are hard to estimate
- PERT answers questions e.g.
  - What is probability of completing project within 20 days?
  - If we want a 95% level of confidence, what should the project duration be?

Where did PERT originate?

PERT history

Was developed in the 1950’s for
the USA Polaris Missile-submarine program

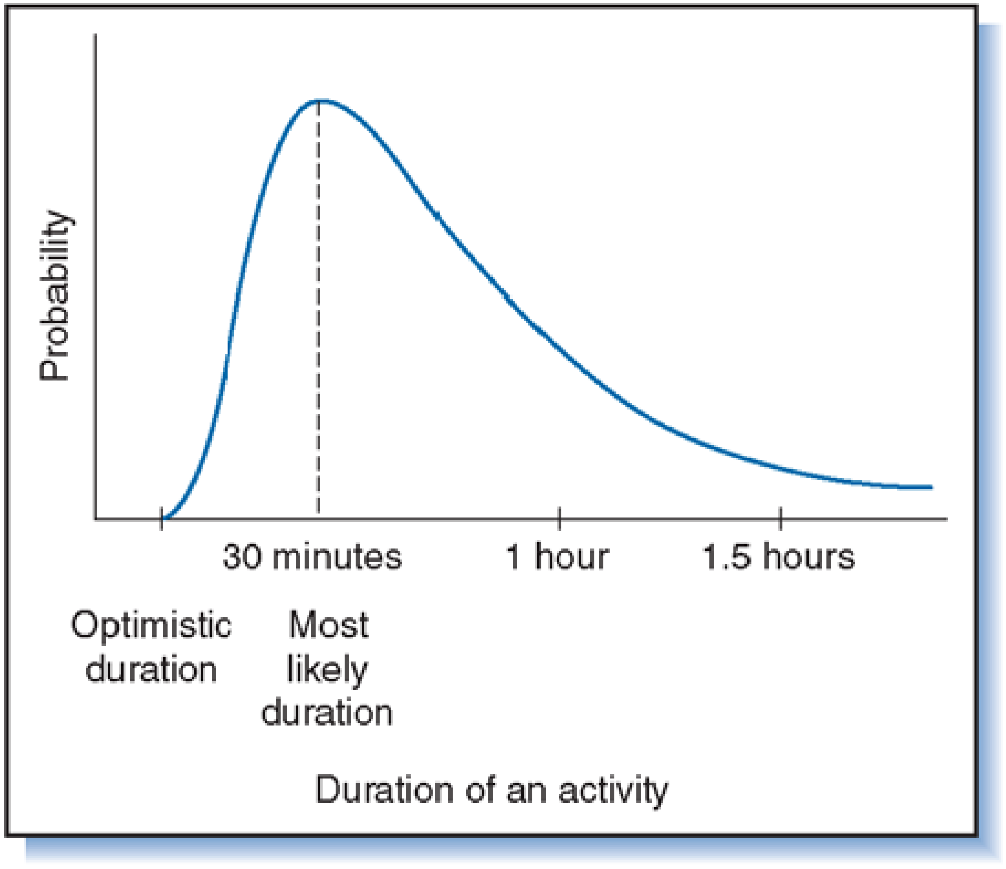
- USA Naval Office of Special Projects
- Lockheed Corporation
  (now Lockheed-Martin)
- Booz, Allen, Hamilton Corporation






PERT Technique

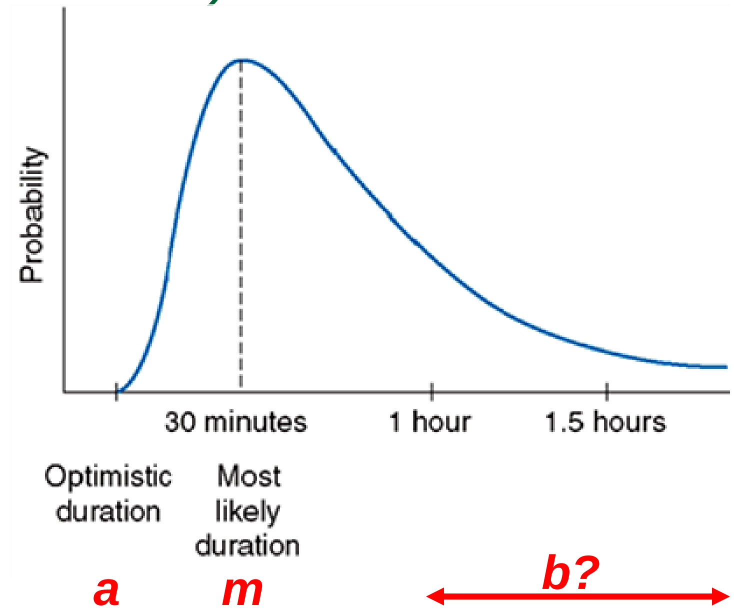
Assume duration of every activity is range of times represented by probability distribution



PERT Technique (Cont'd)

Distribution is based upon three estimates for each activity:

a = optimistic
m = most likely
b = pessimistic



The estimates are presumably based upon experience

What should the pessimistic duration be?

PERT Technique (Cont'd)

Pessimistic duration *b*:

Exclude highly unlikely events e.g.

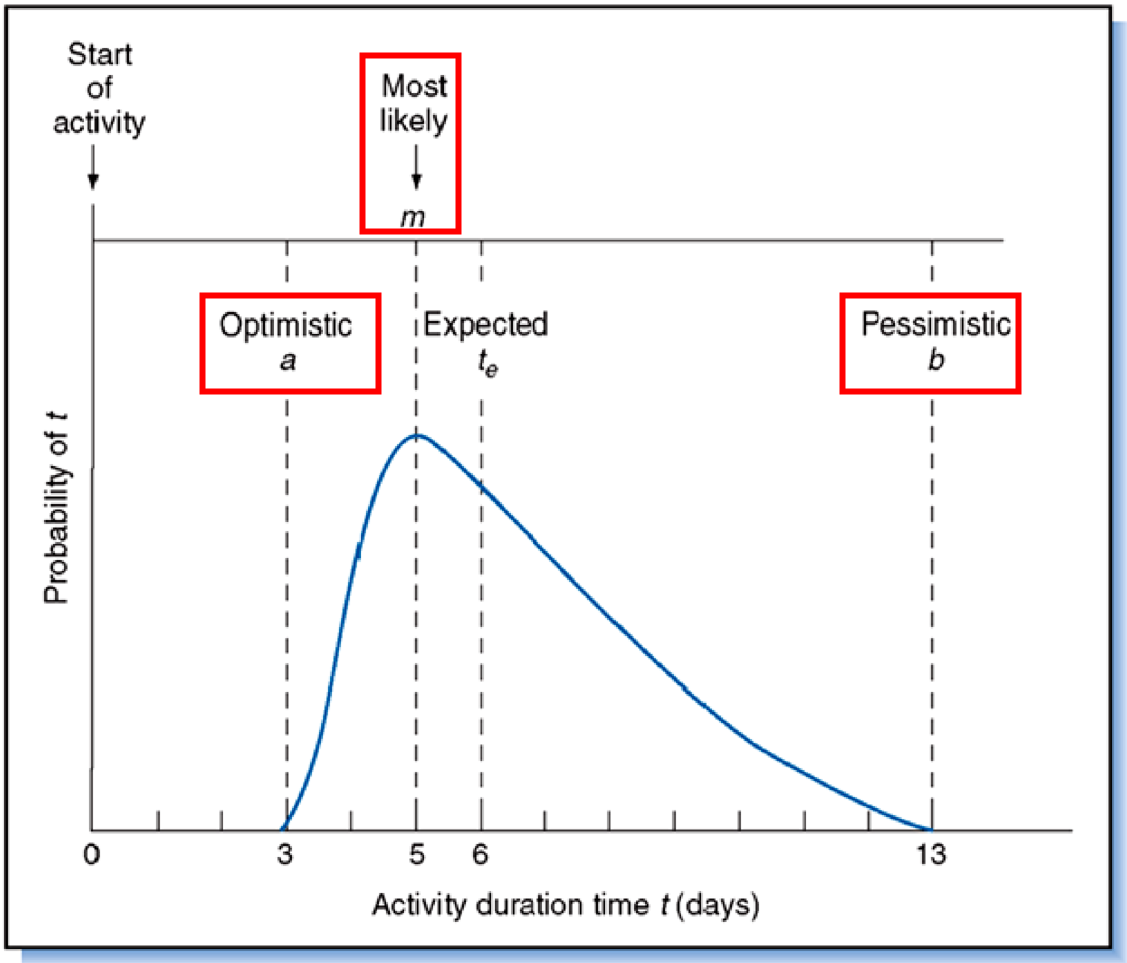
- Earthquakes
- Labor strikes

$\Rightarrow$ Definite cut-off point for *b*

PERT Technique (Cont'd) Page 250

Definite cut-off point for the pessimistic value

Figure 7-11
Estimating activity duration time.



PERT Technique (Cont'd)

Now, given the *a, b* and *m* estimates, for every activity compute expected time t_e

$$t_e = \frac{a + 4m + b}{6}$$

Where *a* = optimistic
m = most likely
b = pessimistic

Example:
Assume *a* = 3, *m* = 6, *b* = 15

Then $t_e = 7$

PERT Technique (Cont'd)

Also, given the a, b and m estimates, for every activity compute the standard deviation, σ

$$\sigma = \frac{b - a}{6}$$

Since $V = \sigma^2$,

$$V = \left(\frac{b - a}{6}\right)^2$$

Example: assume *a* = 3, *m* = 6, *b* = 15

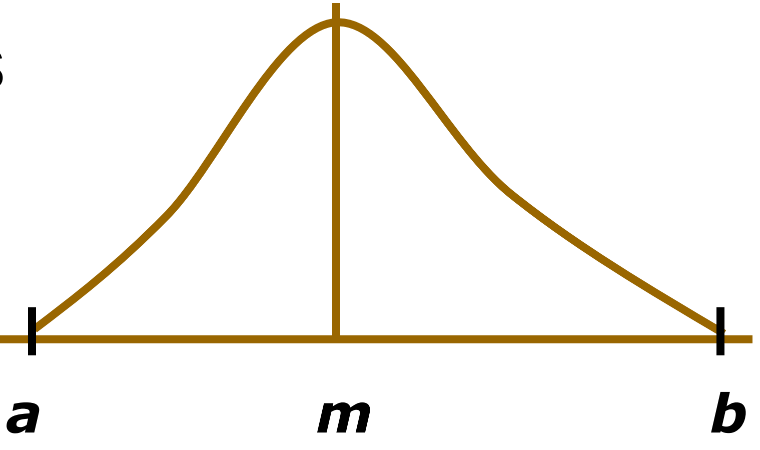
Then σ = 2

PERT Technique (Cont'd)

These formulas are based on assumption that each activity duration conforms to *Beta* distribution (not Normal distribution)

Beta Distribution:

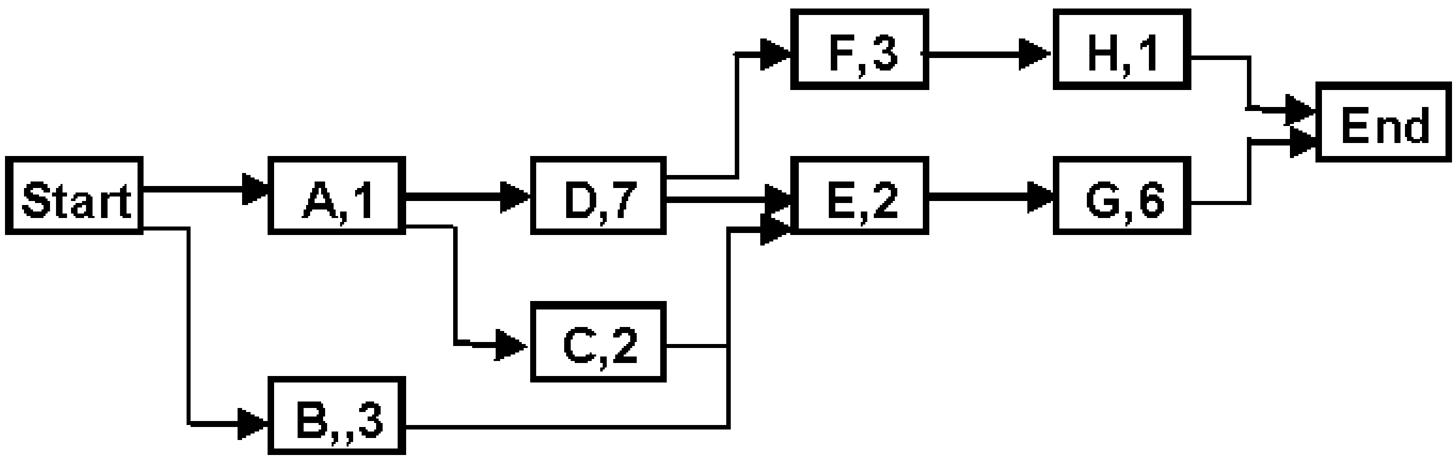
- Not necessarily symmetrical
- Definite cut-off points
- A single peak



PERT Technique (Cont'd)

Step 1:

- For each activity calculate the t_e value $(a + 4m + b)/6$
- Everywhere in network, insert expected time, t_e

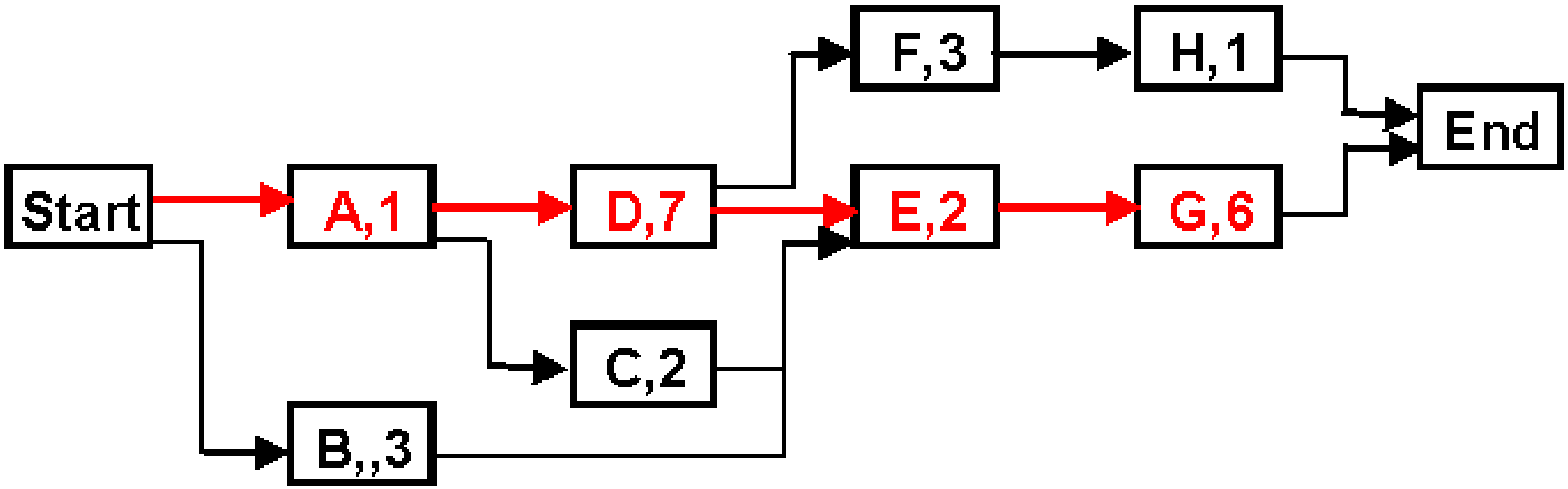


PERT Technique (Cont'd)

Step 2:

Identify the critical path, based on t_e values

CP is A-D-E-G, which indicates expected project completion time is 16 days



What is probability that project will be completed in 20 days?

PERT Technique (Cont'd)

Step 3:

Consider the summative distribution of all activities on the critical path

Assume distribution of *project* completion is normal, not skewed

(justified by the Central Limit Theorem – discussed later)

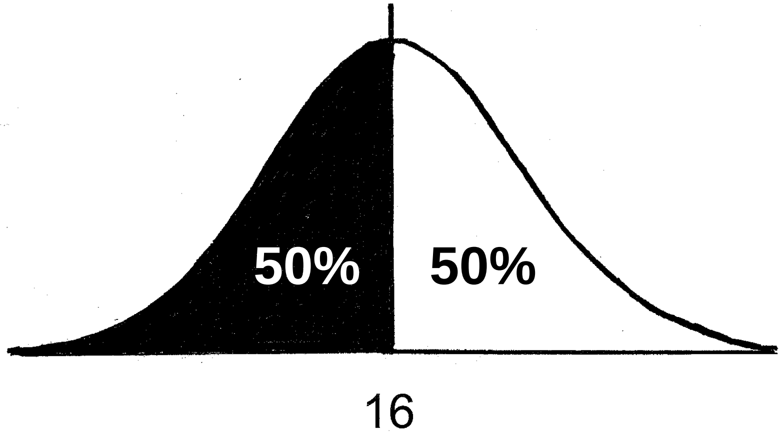
σ = standard deviation of project duration

PERT Technique (Cont'd)

Step 3 (Cont'd)

Consider the summative distribution of all activities on the critical path

An expected project completion date of 16 days means a 50% probability of duration being less than 16 days, (and 50% probability of it exceeding 16 days)

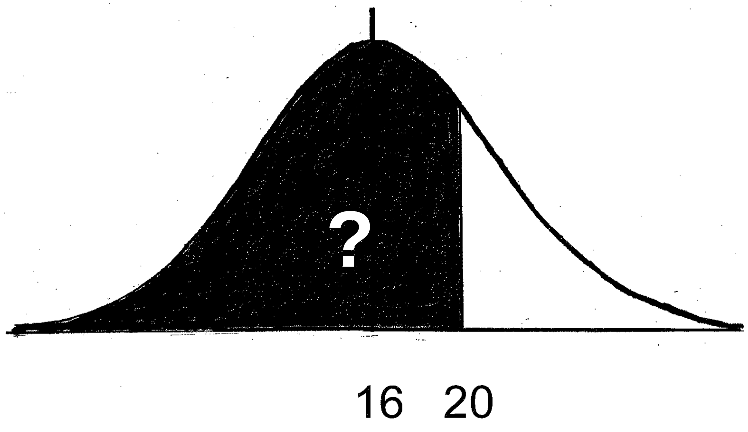


PERT Technique (Cont'd)

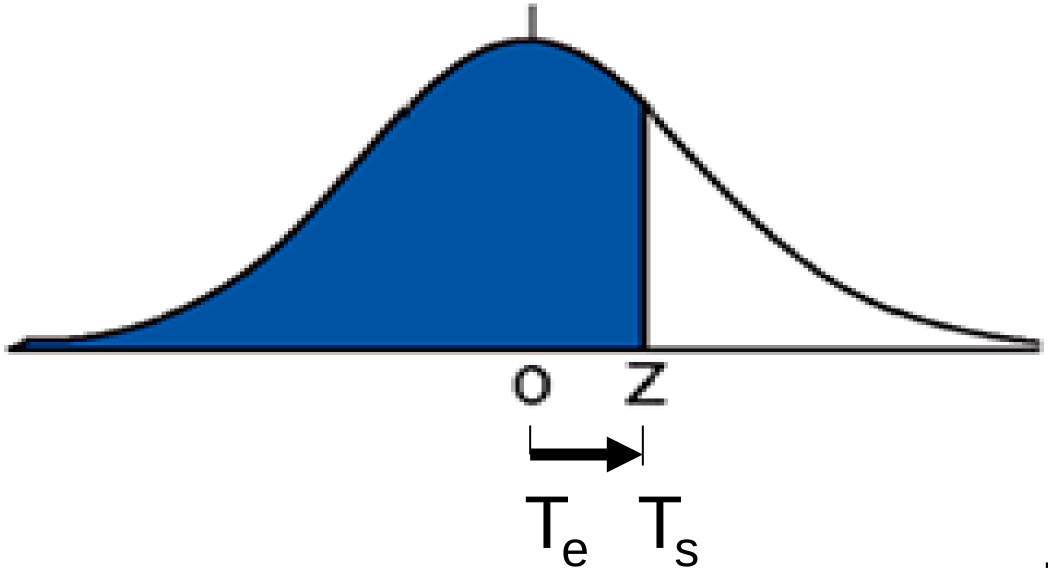
Step 3 (Cont'd):

Consider the summative distribution of all activities on the critical path

To determine the probability of finishing the project within 20 days, compute the area to left of 20 on distribution,
P (x ≤ 20)



PERT Technique (Cont'd)

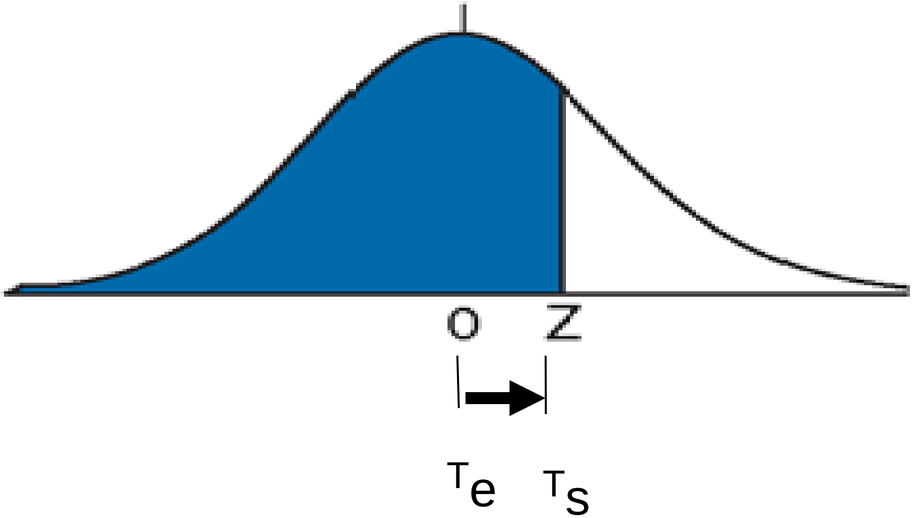


$$Z = (T_s - T_e) / \sigma$$

σ = standard deviation for project

Z = number of standard deviations from mean project duration

Technique (cont'd)



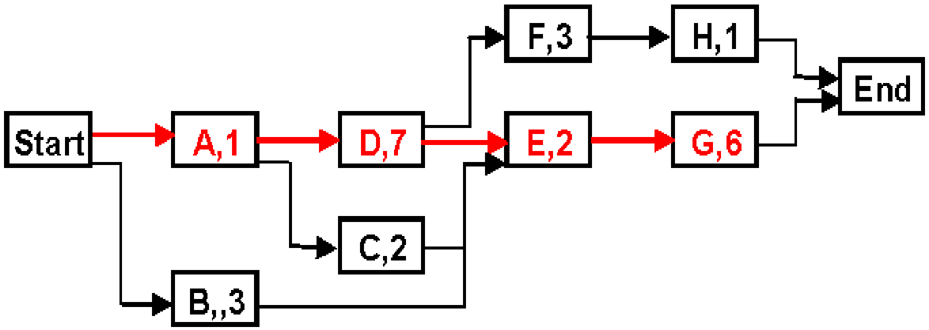
T_e = 16 T_s =20

T_e = expected project duration = $\Sigma\ t_e$

T_s = project completion time of interest

Technique (cont'd)

Step 4:



Compute **T**$_e$, σ, and variance for the critical path

Assume the following:

CP	t_e	σ	σ^2 = V =variance
A	**1**	**1**	**1**
D	**7**	**2**	**4**
E	**2**	**1**	**1**
G	**6**	**1**	**1**
	16 = T_e		**7 = V**

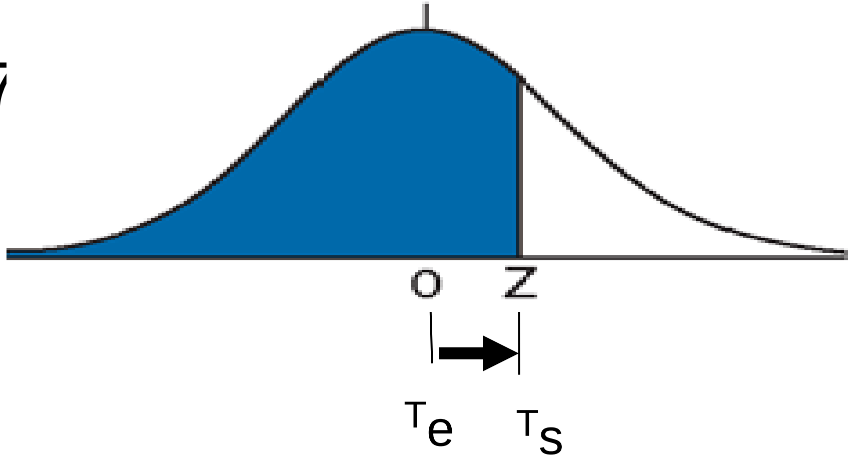
$V_{project} = \sum V_{CP} = \sum\sigma^2 = 7$ (see later why we add up variances)

Technique (cont'd)

Thus, $V_P = \sum\sigma = 7$, so $\sigma = \sqrt{7}$

Compute z–value

$$z = \frac{T_s - T_e}{\sqrt{V_p}}$$



$T_e = 16$ $T_s = 20$

For project duration of 20 days:

$$Z = \frac{T_s - \sum t_e}{\sigma_p} = \frac{20 - 16}{\sqrt{7}} = 1.52$$

Technique (cont'd)

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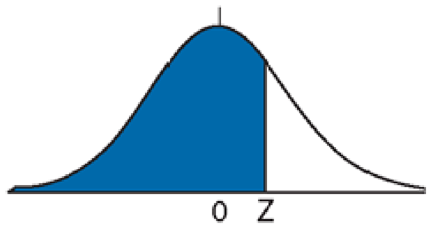
$P(z \leq 1.52) = 0.93$

(approximately 93%. As estimates are used, higher accuracy does not make sense)



Z Value	Probability
0.0	.50
0.1	.54
0.2	.58
0.3	.61
0.4	.66
0.5	.69
0.6	.72
0.7	.76
0.8	.79
0.9	.82
1.0	.83
1.1	.86

Z Value	Probability
1.2	.88
1.3	.90
1.4	.92
1.5	.93
1.6	.95
1.7	.96
1.8	.96
1.9	.97
2.0	.98
2.1	.98
2.2	.99
2.3	.99

Technique (cont'd)

Hence, conclude that there is a 93% probability that the project will be completed in 20 days or less

Summary: The Role of PERT

PERT does *not* reduce project duration

However, it does the following:

1. Given a network with estimates a, m, and b as well as a value for project duration, it provides a probability figure for finishing on time
2. Alternatively, given a network with estimates a, m, and b as well as a desired level of confidence (probability figure, say 99%), it can calculate a project duration that corresponds with the level of confidence
3. It provides insight in the effect of variability of activity duration on the critical path

Interpretation

- Now the question is: How *confident* are we in the 93% estimate? How much do you *trust* that estimate?
- 93% is high percentage. So, can we be very confident that project will be finished in less than 20 days?

Interpretation

Answer:

1. Confidence in estimates a, m, and b

    - If estimates are based upon experience backed by historical data, maybe we can believe the 93% estimate
    - If a, m, and b are guesses, be careful! If any of these estimates are substantially incorrect, the computed % will be meaningless

2. The method only considers the critical path and is misleading when near-critical paths could become critical

PERT only considers the critical path

There are often “near critical” paths

Figure 7-12

PERT network with expected activity times and activity variances.

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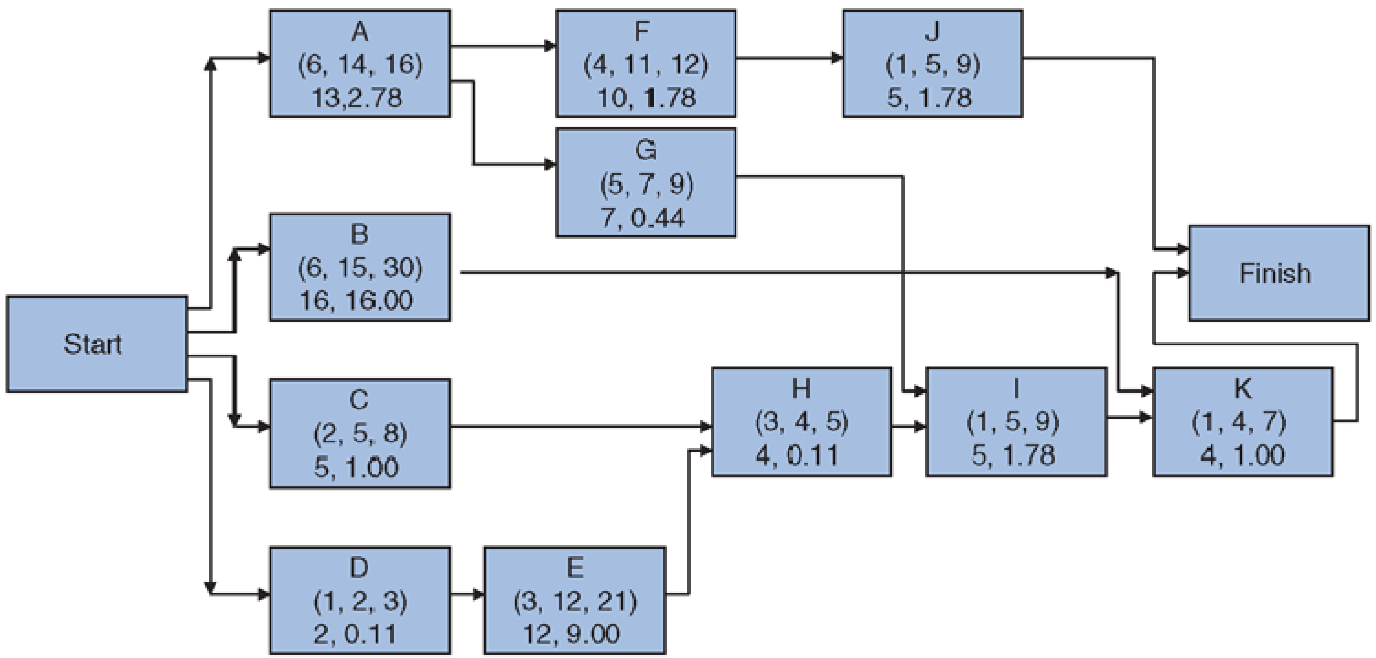

Path	T_e	t_e	V_p	V
A–F–J		28**		6.34
B–K		20		17.00
A–G–I–K	T_e =	29*	V_p =	6.00
C–H–I–K		18		3.89
D–E–H–I–K		27**		12.00

* Critical path
**“Near”critical paths

Key

Activity
(a, m, b)
t_e, V

Shortcoming:
PERT only considers the critical path

PERT only considers the critical path and is misleading when near-critical paths could become critical

Merge-point bias:

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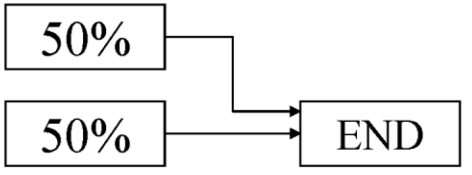

Two paths merging, each 50% chance of being on time

25% chance of finishing on time (or early)

Merge-point bias

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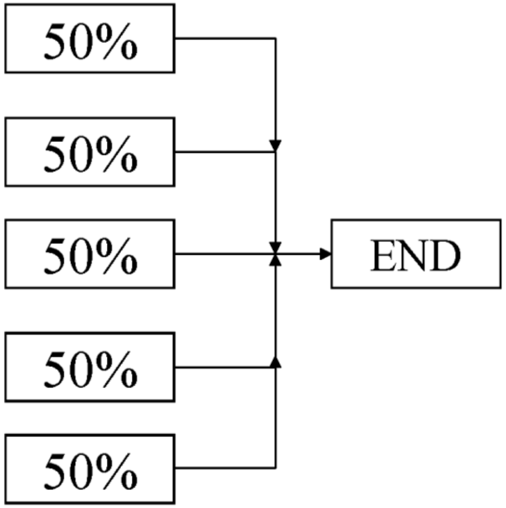

Five paths merging, each with 50% chance of being on time

Probability of project finishing on time = ($0.5^5 \approx .03$ or 3%)

c.a. 3% chance of finishing on time

Non-critical paths and merge-point bias

The problem of near-critical paths that could become critical and merge-point bias can be addressed by **Monte-Carlo simulation** of the entire network

Times for project critical and non-critical *activities* are randomly selected from probability distributions

The critical path is computed from these times

The procedure is repeated many times to generate a distribution diagram for the *project*

Non-critical paths and merge-point bias

Merge-point bias addressed by **Monte-Carlo simulation** of the network

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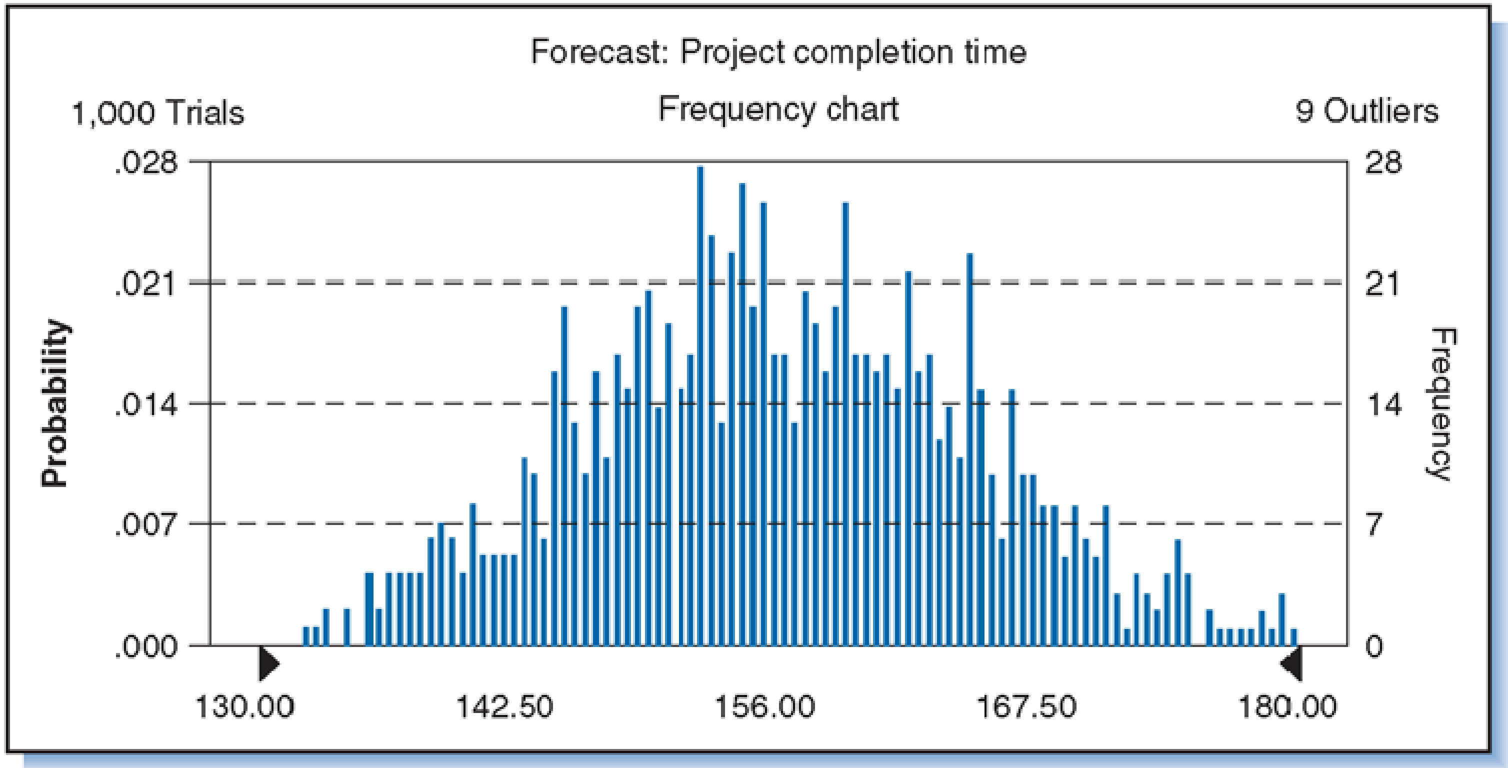
Figure 7-14
Crystal Ball simulation results for project completion times.



Other Shortcomings of PERT

- Assumes that a successor will start immediately when predecessors completed, also when an activity is completed earlier than indicated on the schedule

- PERT technique can provide false confidence

- Expecting high probability of project completion, managers let their guard down!

Risk of Adding Up Most Likely Values

In practice many managers simply add up activity durations on the critical path and are not aware of the risk of doing so

(They also add up most likely cost figures)

Let’s look at the risk of this deterministic approach (consider only the critical path of a project)

Risk of Adding Up Most Likely Values

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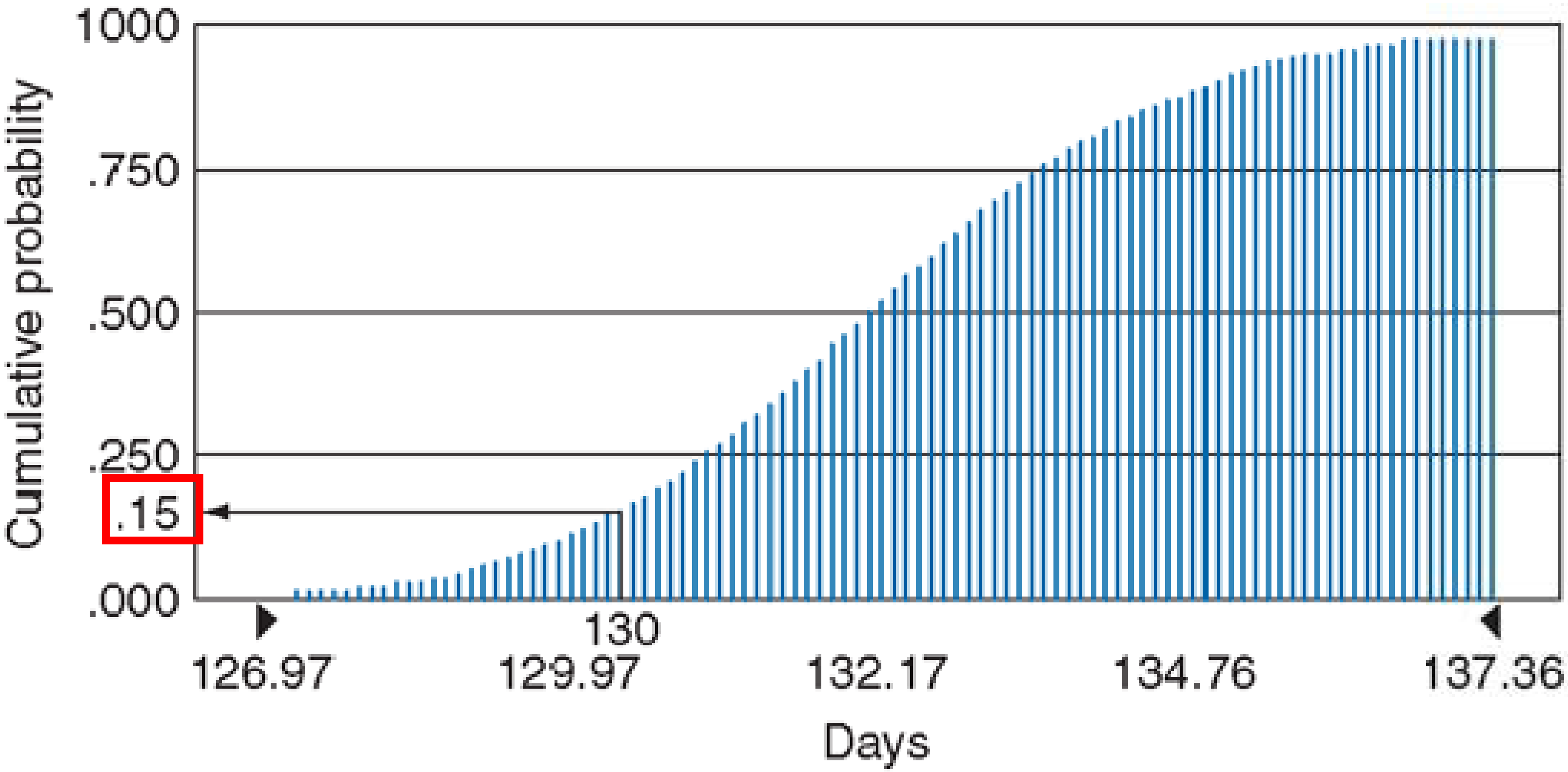
Activities on critical path	Optimistic Duration *a*	Most Likely Duration *m*	Pessimistic Duration *b*
A	10	12	15
B	14	15	17.5
C	18	20	22
D	12	13	14.5
E	22	24	27
F	14	15	17
G	13	14	15.5
H	16	17	19
Sum of most likely durations		130	

What is the risk of committing to 130 days?

Risk of Adding Up Most Likely Values

Result of simulating the critical path: **Page 258**



15% chance of finishing within 130 days

Risk of Adding Up Most Likely Values

Simulation indicates: 85% chance of a commitment on 130 days getting you into trouble

This does not even take into account:

- Non-critical paths becoming critical
- Behavioral aspects

$\Rightarrow$ Less than 15% chance of delivering on time

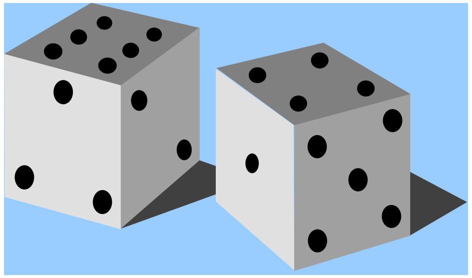
The Theoretical Basis of PERT: The Central Limit Theorem

Probability distributions for activities are skewed.

So, why did we assume a normal distribution for project duration?

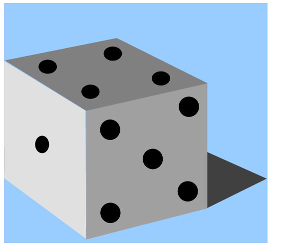
Summarizing of distributions are involved

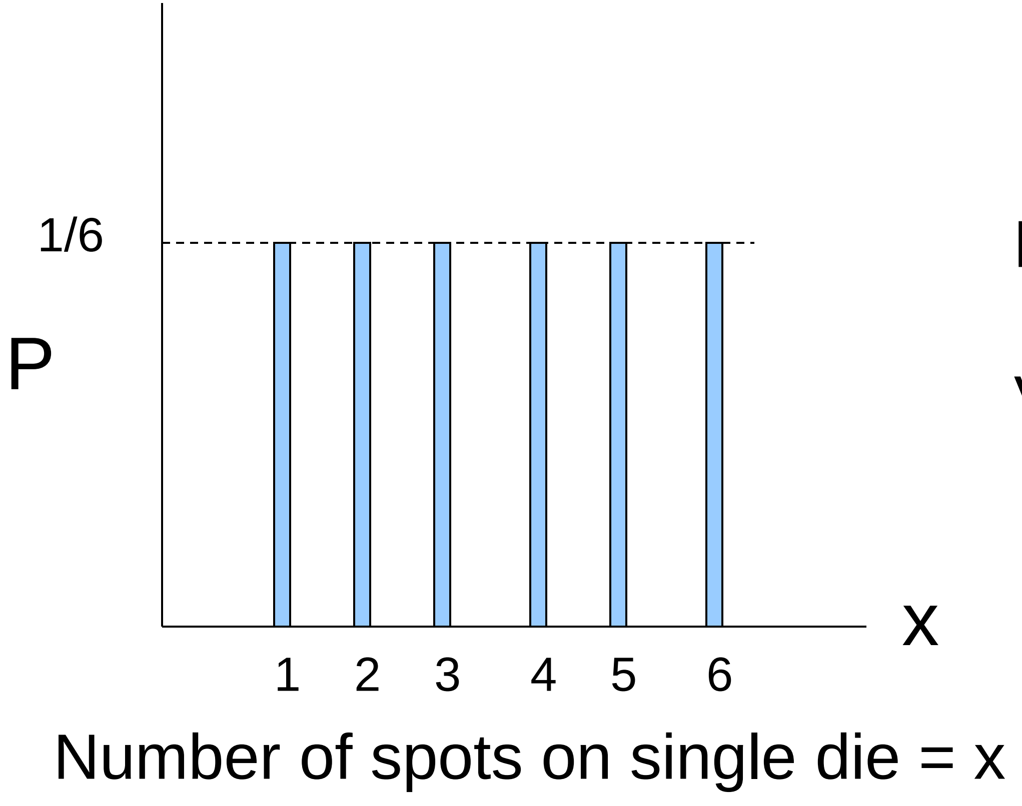
Consider another example of summarizing distributions: throwing of dice:



The Central Limit Theorem

Distribution for throwing one die:

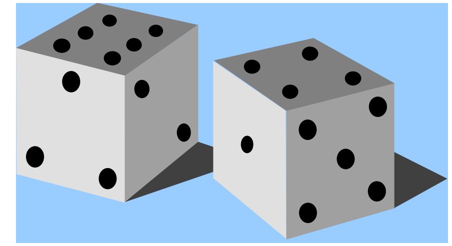




Mean of x = 3 ½

Variance of x = 2 $^{11}/_{12}$

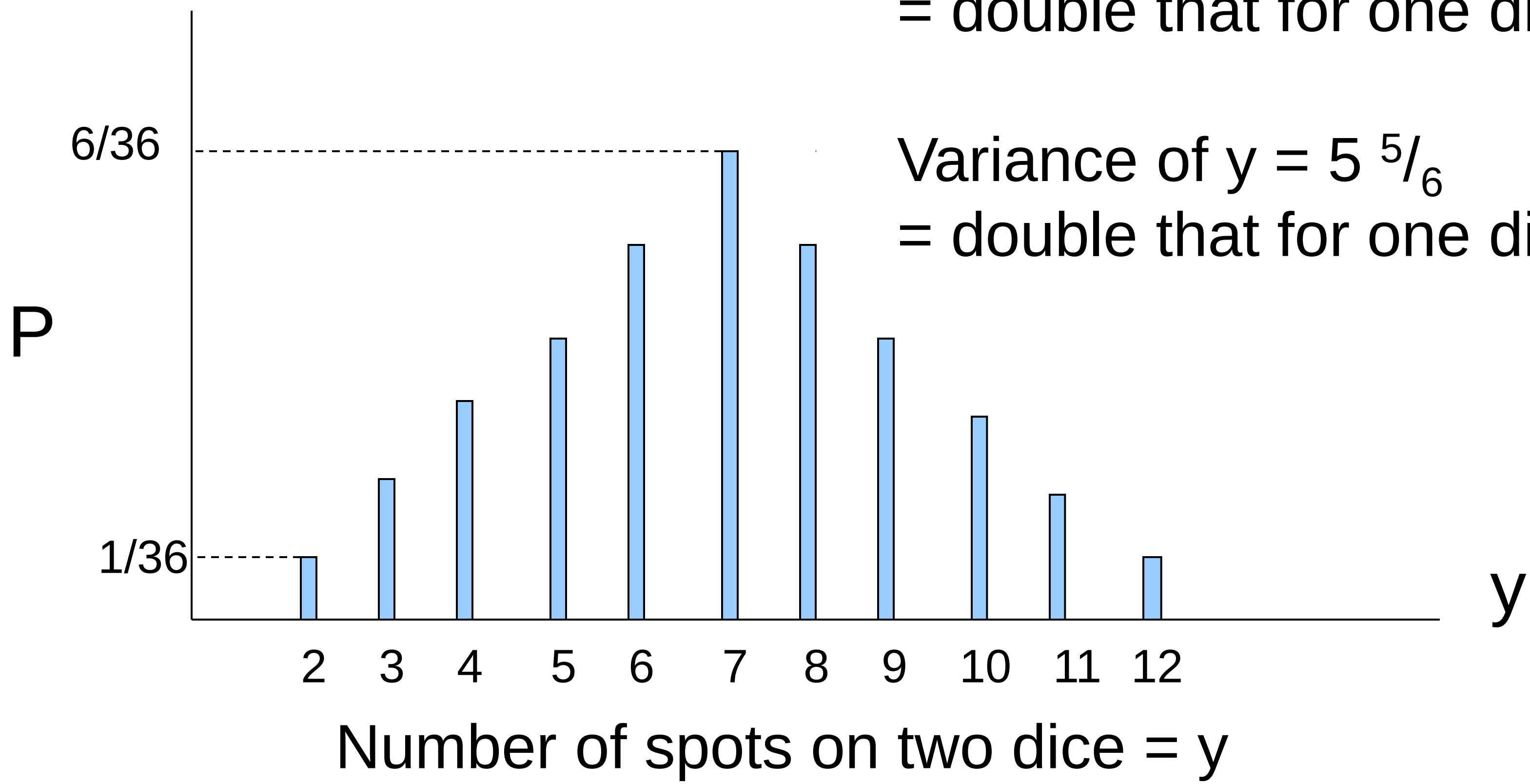
The Central Limit Theorem



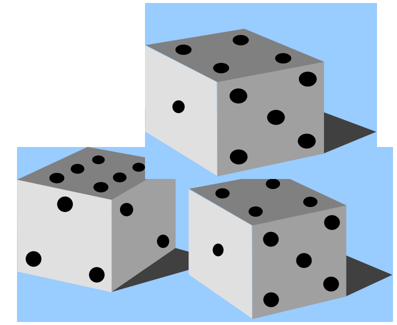
Distribution for throwing two dice

Mean of y = 7
= double that for one die

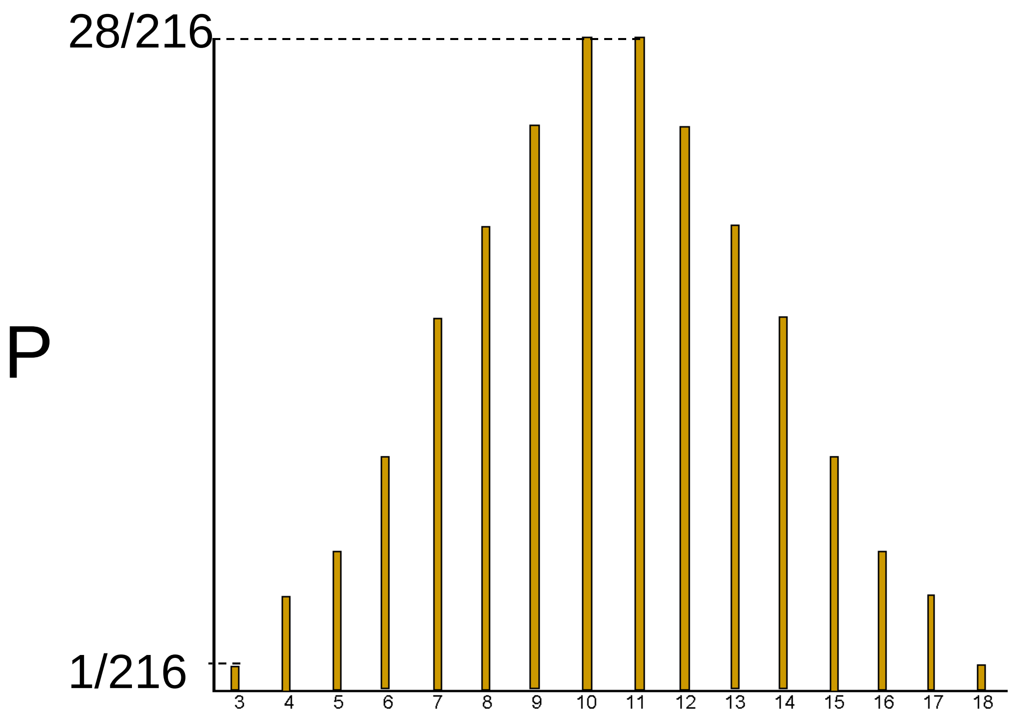
Variance of y = 5 $^{5}/_{6}$
= double that for one die



The Central Limit Theorem



Distribution for throwing three dice:



Mean of z = 10 ½
= 3 x that for one die

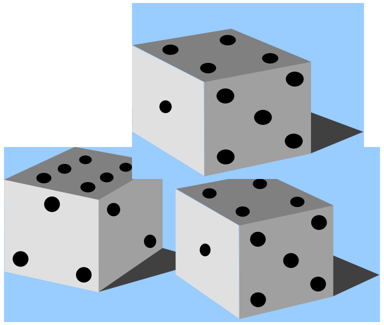
Variance of z = 8 ¾
= 3 x that for one die

Note the bell shape

z

Number of spots on three dice = z

The Central Limit Theorem



Note that:

1. The more distributions we "add together", the closer the summated distribution gets to the bell shape of the normal distribution

2. The mean of the summated distribution

   = the sum of the individual distributions

3. The variance of the summated distribution

   = the sum of the individual distributions

The Central Limit Theorem

Provided that:

- *n independent* tasks are to be performed in sequence (e.g. on a critical path)

- *n* is a relatively large number (in practice for PERT even 4 or 5)

The Central Limit Theorem

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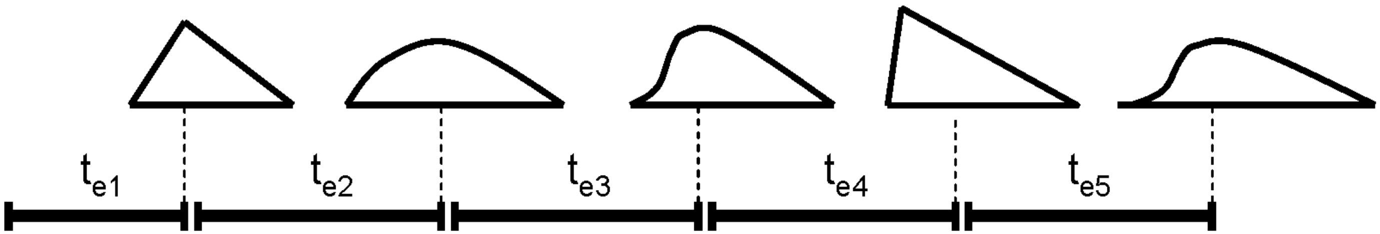
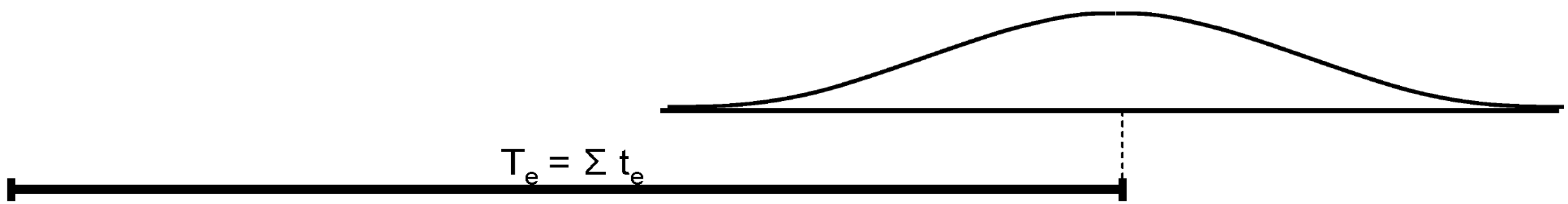


(a) Project with 5 activities in sequence, each with a specific skewed duration distribution



(b) Project duration for 5 activities in (a) with distribution more or less normal

The Central Limit Theorem

1. The distribution of the sum is approximately normal
2. The mean of the sum = the sum of individual means
3. The variance of the sum = the sum of the individual variances

This justifies why we could:

1. Assume a normal distribution for project duration (in Step 3)
2. Add up variances of individual activities (Step 4)